IMPROVED CODES FOR SYNCHRONIZATION ERROR CORRECTION ON THE BPMR CHANNEL

Phan-Minh NGUYEN, Marc A. ARMAND and Tong WU

National University of Singapore, Singapore

I. INTRODUCTION

We study the data recovery problem for bit-patterned media recording (BPMR) with written-in errors, in particular, bit insertions and deletions. The Davey-MacKay (DM) concatenated code construction is applied to combat written-in errors with a twist by dropping the watermark string. This is made possible through the proposed inner code design procedure. The overall system performance with the proposed inner codes on a staggered BPMR channel corresponding to an areal density of 10 Tb/in² is presented.

II. THE IMPROVED INNER CODES

Partially capturing a key property of the write channel model that synchronization drifts are at most ± 1 , we propose that a good binary inner code *C* of size *Q* and length *n* should minimize the quantity $F(C) = \sum_{x \in C} F(C, x)$ where F(C, x) is defined to be

$$\frac{1}{Q} \left[\sum_{\boldsymbol{y} \in C \setminus \{\boldsymbol{x}\}} P(\boldsymbol{y}|\boldsymbol{x}) + \frac{1}{2} \frac{p_i p_d^2 + p_i^2 p_d}{p_i^2 + p_i p_d + p_d^2} \left(\sum_{\boldsymbol{r} \in \{0,1\}} \sum_{\boldsymbol{y} \in C \setminus \{\boldsymbol{x}\}} P([\boldsymbol{y}_2^n, \boldsymbol{r}]|\boldsymbol{x}) + \sum_{\boldsymbol{r} \in \{0,1\}} \sum_{\boldsymbol{y} \in C \setminus \{\boldsymbol{x}\}} P([\boldsymbol{r}, \boldsymbol{y}_1^{n-1}]|\boldsymbol{x}) \right) \right]$$

 p_i and p_d are the insertion and deletion probabilities as in [1], y_a^b is the sequence of bits in codeword y corresponding to indices a to b, and [q, t] denotes the concatenation of vectors q and t. The probability metric $P(\cdot | \cdot)$ is computed in a similar manner to [1], noticing that drifts outside ± 1 are not allowed. With the given objective function, finding the global minimum is generally intractable. We therefore propose a computationally efficient greedy-strategic procedure to obtain a suboptimal solution as follows. C is first initialized to $GF(2)^n$. Then, we repeat finding $x \in C$ such that F(C, x) is largest and assigning $C \coloneqq C \setminus \{x\}$, until the size of C is reduced to Q.

III. SIMULATION RESULTS

Fig. 1 shows the frame error rate (FER) plots for different inner coding schemes of rate 4/5 and 8/9 with the same outer low-density parity-check (LDPC) code for each rate, on the write channel of [5] for various insertion/deletion probabilities (which are set to be equal). Inner codes considered include the DM sparse codes, the single parity-check (SPC) codes and our improved inner codes. Evidently, the improved codes with no watermark string outperform the other schemes. Moreover, when a random watermark string is added, their performance worsens. The rate-8/9 inner coding schemes considered in Fig. 1 are further employed on a staggered BPMR write-read channel, which is obtained by concatenating the write channel of [5] to the single-track staggered BPMR read channel of [3]. The hexagonal array of island distribution proposed in [4] for areal density of 10 Tb/in² is considered and the signal-to-noise ratio (SNR) of the read channel is set to 12 dB. The FER performance is presented in Fig. 2. As expected, the improved inner code without watermark yields the best FER performance.

Marc A. Armand Dept. of Electrical & Computer Engineering National University of Singapore 4 Engineering Drive 3 Singapore - 117583 E-mail: eleama@nus.edu.sg Tel: +65-6516-8889

- 1) P.-M. Nguyen, M. A. Armand, and T. Wu, *IEEE Commun. Lett.*, **17**, no. 9, 1830-1833, (2013).
- 2) T. Wu, M. A. Armand and J. R. Cruz, IEEE Trans. Magn., 50, no.1, 1-11, (2014).
- 3) W. Chang and J. R. Cruz, IEEE Trans. Magn., 47, no. 10, 2551-2554, (2011).
- 4) Y. Wang and R. H. Victora, *IEEE Trans. Magn.*, 49, no. 10, 5208 -5214, (2013).
- 5) T. Wu and M. A. Armand, *IEEE Trans. Magn.*, 49, no.1, 489-495, (2013).



Fig. 1 FER plots corresponding to different inner coding schemes on the write channel. n = 5 (resp., n = 9) for rate 4/5 (resp., 8/9). "Watermark 2/2ⁿ" refers to the watermark string design from [2]. "Random watermark" refers to a randomly generated watermark. The substitution error rate is 10^{-3} . Burst errors are not considered.



Fig. 2 FER performance corresponding to different inner coding schemes of rate 8/9 on the write-read channel with areal density = 10 Tb/in² and read channel SNR = 12 dB. Sector length is 5112. The soft-input inner decoder of [2] is used. The extended multi-track detection scheme of [3] is employed to effectively mitigate the 2D intersymbol interference. The substitution error rate is 10^{-3} . Burst errors are not considered.